TOPOLOGY

SAMPLE * PROBLEM

Dan

Section and problem number

Name

§1.9 Problem x1

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Problem statement

Let $A_1, A_2, \cdots, A_n$ and $B_1, B_2, \cdots, B_n$ be sets, when n is a positive integer. Show if
$A_1 \cap B_1 = \emptyset$, $A_2 \cap B_2 = \emptyset$, $\cdots$, $A_n \cap B_n = \emptyset$
then $(A_1 \cup A_2 \cup \cdots \cup A_n) \cap (B_1 \cap B_2 \cap \cdots \cap B_n) = \emptyset$.

Solution written in complete sentences, with correct format.

Proof: Suppose that, contrary to the stated conclusion, $(A_1 \cup A_2 \cup \cdots \cup A_n) \cap (B_1 \cap B_2 \cap \cdots \cap B_n) \neq \emptyset$. Then there must be an element x such that

$$x \in A_1 \cup A_2 \cup \cdots \cup A_n \qquad (1)$$

and

$$x \in B_1 \cap B_2 \cap \cdots \cap B_n. \qquad (2)$$

By (1), for some integer k with $1 \leq k \leq n$,
$$x \in A_k.$$
By (2), $x \in B_j$ for every j, and in particular, $x \in B_k$ for the same k mentioned before. That is,

$$x \in A_k$$

and

$$x \in B_k.$$

But then $x \in A_k \cap B_k$, contradicting the original assumption that each $A_j \cap B_j = \emptyset$. In particular, for at least one integer, k, we have shown $A_k \cap B_k \neq \emptyset$.

Since assuming $(A_1 \cup A_2 \cup \cdots \cup A_n) \cap (B_1 \cap \cdots \cap B_n) \neq \emptyset$ leads to a contradiction, this assumption must itself be false. This shows
$$(A_1 \cup A_2 \cup \cdots \cup A_n) \cap (B_1 \cap B_2 \cap \cdots \cap B_n) = \emptyset$$
as desired.