Two Proofs of 1st Hint on Exercise 6

We want to show that Rev $f(x) = x^n f(1/x)$ when f is a polynomial of degree n and $f(0) \neq 0$.

Proof 1 (three dot version):

Let f be an arbitrary polynomial of degree n, with $f(0) \neq 0$. Then we can express f in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where the coefficients a_j are constants, and $a_0 = f(0) \neq 0$. Then

$$x^n f(1/x) = x^n f(x^{-1}) = x^n\left(a_n x^{-n} + a_{n-1}x^{-(n-1)} + \cdots + a_1 x^{-1} + a_0\right)$$
$$= a_n + a_{n-1}x + \cdots + a_1 x^{n-1} + a_0 x^n.$$

But by definition, this is equal to Rev $f(x)$.

Proof 2 (Sigma notation version):

Let f be an arbitrary polynomial of degree n, with $f(0) \neq 0$. Then we can express f in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = \sum_{j=0}^{n} a_j x^j$ where the coefficients a_j are constants, and $a_0 = f(0) \neq 0$. Then by definition,

$$\text{Rev } f(x) = a_n + a_{n-1}x + \cdots + a_1 x^{n-1} + a_0 x^n = \sum_{j=0}^{n} a_j x^{n-j}.$$

Using algebra,

$$x^n f(1/x) = x^n f(x^{-1}) = x^n \sum_{j=0}^{n} a_j x^{-j} = \sum_{j=0}^{n} a_j x^n x^{-j} = \sum_{j=0}^{n} a_j x^{n-j}.$$

Thus we see that Rev $f(x)$ and $x^n f(1/x)$ are equal.