User's Guide for Primus-Logistic-Models.xlsx
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1. Overview
This spreadsheet file is intended to provide an easy way for students to define and study applications of refined discrete logistic growth difference equations, as well as continuous logistic growth functions. For the discrete models, after entering parameter values, the user will be immediately provided with a table of sequence terms and a graph. The user can also enter data values to which the model is intended to fit. Then trial-and-error variation of the parameters can be used to approximately optimize the agreement of the model to the data. A similar process can be followed with the continuous logistic model page.

2. Models
The refined logistic difference equation is $a_{n+1} = \frac{a_n}{ma_n+b}$, with constant parameters m and b. This produces a sequence for which the reciprocal growth factors a_n/a_{n+1} depend linearly on a_n. The terms of the sequence lie on the graph of a continuous logistic function of the form $y = \frac{A}{1+B \cdot R^x}$, where A, B, and R are constant parameters.

3. Pages
The excel file has seven pages (or worksheets) as detailed in the following table. The pages are accessed using tabs located at the bottom of the excel window.

Page name	Description
diffeqn	Without data, enter parameters for the refined logistic difference equation, observe graph and a table of sequence terms.
diffeqn + datapoints	Enter both data and parameter values, observe agreement between model and data. Use trial-and-error parameter variation to fit model to data.
find m and b	Estimate initial refined logistic growth parameters by fitting a linear model to reciprocal growth factors in data.
logistic function	Enter data and parameters for a continuous logistic function. Use trial-and-error parameter variation to fit model to data.
Find A, B, R	Compute parameters for a logistic function from estimates of the inflection point and the *y*-intercept.

4. General Information About Spreadsheet Pages
In the upper left-hand corner of most pages the user can view instructions for that page. Generally, users are intended to enter data only in yellow cells. On each page where data and a model are being compared, the average error is displayed. This is actually the average of the absolute differences between data values and the corresponding model values. It appears either directly above or below the graph, and is labeled *Avg. Error*, or for continuous logistic models, *Average Absolute Error*.

5. Additional Information About Specific Pages

find m and b. To formulate a **refined logistic growth** sequence approximating a set of data, it is necessary to define a linear model for reciprocal growth factors. This is equivalent to defining an inverse-linear model for the growth factors, in the form $r = 1/(mp + b)$, and is supposed to estimate the *reciprocal* growth factor for a particular data value as a linear function of the data value. Given data values $d_0, d_1, d_2, \cdots$, the corresponding reciprocal growth factors are $s_0 = \frac{d_0}{d_1}$, $s_1 = \frac{d_1}{d_2}$, $s_2 = \frac{d_2}{d_3}, \cdots$.
Plotting the points $(d_0, s_0), (d_1, s_1), (d_2, s_2), \cdots$, we attempt to find a straight line that is as close as possible to all the points. The *find m and b* page assists with this process. It computes and tabulates the reciprocal growth factors for a given set of data points, and shows the points $(d_0, s_0), (d_1, s_1), (d_2, s_2), \cdots$, on a graph. Then the user enters parameters defining a linear equation, views the corresponding line on the graph and uses trial-and-error to obtain as close a fit as possible to the plotted points. The page also automatically calculates the *m* and *b* parameters for a refined logistic growth difference equation based on the approximating line. These in turn can be used to define the refined logistic growth sequence on the *diffeqn + datapoints* page.

Find A, B, R. This page does not have instructions in the upper left-hand corner, because the interaction is very simple. The user enters numerical values in three yellow cells and observes the computed values of *A*, *B*, and *R*. An explanation at the top of the page tells what values to enter in the yellow boxes and how *A*, *B*, and *R* are computed by the spreadsheet. This page can be used in connection with the *logistic function* page. Entering data on the latter page, the user visually estimates the inflection point and the *y* intercept for an appropriate logistic curve. This provides the three inputs to the *Find A, B*, and *R* page. Then the resulting values of *A*, *B*, and *R* can be entered on the *logistic function* page as a first approximation of a logistic function fitting the entered data points.

6. Copying Cells in Excel
Values entered or computed on one page can be copied to another page. The general procedure is to select the cells to be copied, execute the *copy* function, select the cell where the copy is to be placed, and then execute the *paste* function. For a column of entries, such as input data values, select all the entries, *copy*, then select the destination cell for the first data value, and *paste*. For example, if data have been entered on the *diffeqn + datapoints* page, they can be copied to the *find m and b* page. Then, once values of *m* and *L* have been computed, they can be copied back to the appropriate cells of the *diffeqn + datapoints* page.