A Social Security Math Problem

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(Go to home page for links to slides and an animated graph.)

Overview

- Full benefits available at age 66
- If initiation delayed, monthly benefit is higher
- Delay means foregoing some initial payments
- What is the breakeven point?
- How does the breakeven point depend on the length of the delay?

Specifics

- Let E be the amount of full monthly benefit at age 66
- Delaying payment for k months increases monthly benefit by $2k/3$ %
- Benefit is then $E(1 + 2k/300) = E(1 + k/150)$
- By age 66 + n months will have received

$$T = (n - k)E(1 + k/150)$$

- With no delay, $k = 0$ and $T = nE$

Example

Say $k = 12$. We want total with a 12 month delay to equal the total with no delay

$$(n - 12)E(1 + 12/150) = nE$$

$$(n - 12)(1.08) = n$$

$$0.08n = 12(1.08)$$

$$n = 12(1.08)/0.08$$

$$= 162$$

- Conclusion: after delaying for 12 months, we have to receive 150 payments to break even
- Question: how does the break even point vary with k?

Break Even Analysis

- For $k > 0$, want total with a k month delay to equal the total with no delay

$$(n - k)E(1 + k/150) = nE$$

$$n(1 + k/150) - k(1 + k/150) = n$$

$$n(1 + k/150) - n = k(1 + k/150)$$

$$nk/150 = k(1 + k/150)$$

$$n/150 = 1 + k/150$$

$$n = 150 + k$$

- Conclusion: break even point is always reached after 150 payments, independent of k
- That is, after receiving payments for 12.5 years

Does That Make Sense?

- The longer the delay, the greater the increase in payments
- But you always break even after receiving exactly the same number of payments
- That doesn’t seem right
- What is really going on?

Rationale

- WLOG, assume $E = 1$
- Each payment received is $1 + k/150$
- We get an *extra* $k/150$ with each payment
- We missed k payments, so k dollars
- Have to make up those k dollars by adding up the *extra* amounts of $k/150$
- After 150 payments we will have made up k dollars

Extension

- Waiting k months versus waiting $j > k$ months
- Break even point: $(n - k)$ payments at the k-month-delay rate equals $(n - j)$ payments at the j-month-delay rate.
- Once you start receiving payments after the longer delay, how long will it take to break even?
- Answer: $150 + k$ (so $n = 150 + k + j$)
- Example: Compared to a 12 month delay, any longer delay breaks even after 162 payments at the higher rate.

Greedy Analysis

- Goal: maximize total of payments received
- Assume death occurs at age 66 + m months
- With a k month delay, receive $m - k$ payments of $(1 + k/150)$
- Total: $T = (m - k)(1 + k/150)$
  $= (m - k)(150 + k)/150$
- Quadratic in k with roots m and -150
- Max occurs at $k = (m - 150)/2$

Greedy Analysis, cont.

- Max occurs at $k = (m - 150)/2$ (vertex)
- Domain for k is [0,48]
- If $m \leq 150$, impossible to reach breakeven point for any $k > 0$, so take $k = 0$
- The vertex is inaccessible if

  $$(m - 150)/2 > 48 \iff m > 246$$

  In this case take $k = 48$
- For $m \in [150 , 246]$, max occurs at $k^* = (m - 150)/2$

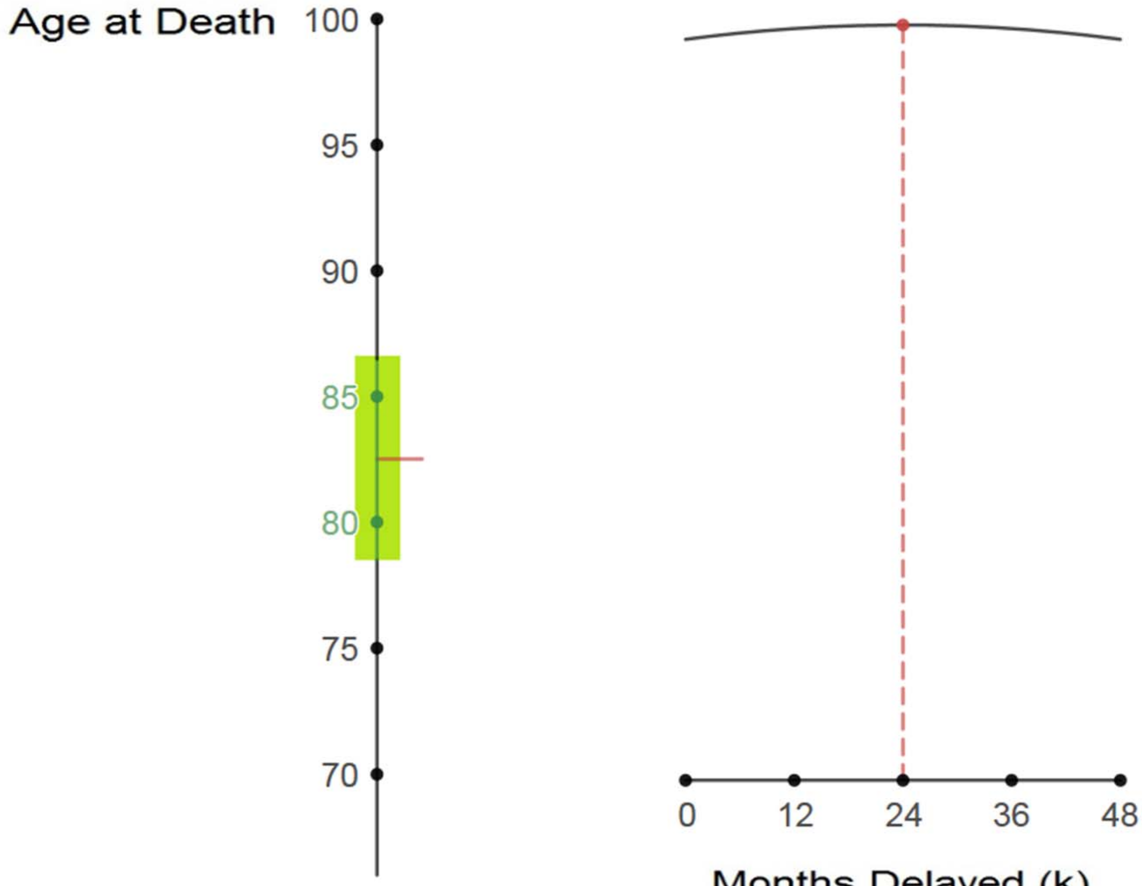
Translation

- If death occurs at or before age 78.5, don't delay starting ss payments
- If death occurs at or after age 86.5, delay starting ss payments for the maximum 4 years
- Otherwise, age at death is $z \in [78.5 , 86.5]$
- Optimal plan: delay by $6(z – 78.5)$ months
- Example: $z = 80$, delay by $6 \cdot 1.5 = 9$ months
- Example: $z = 85$, delay by $6 \cdot 6.5 = 39$ months
- Great analysis if you can predict how long you will live!

Geometry

- Total earnings function $T(k)$ is a family of parabolas, parameterized by m
- On each parabola, restrict $k \in [0 , 48]$
- Varying m moves the vertex $(k^*, T(k^*))$ through this interval
- $k^* \leq 0 \Rightarrow T$ decreases, max at left endpoint
- $k^* \geq 48 \Rightarrow T$ increases, max at right endpoint
- Otherwise, max at k^*

Desmos Animation



https://www.desmos.com/calculator/vanovxjhs8